On Optimization Algorithms for the Design of Multiband Cognitive Radio Networks

Ankita Panwar*, Piyush Bhardwaj*, Onur Ozdemir†, Engin Masazade‡,
Chilukuri K. Mohan‡, Pramod K. Varshney‡, and Andrew L. Drozd†
*Indian Institute of Technology, Kanpur, India
†ANDRO Computational Solutions, LLC, 7902 Turin Road, Rome, NY 13440, USA
‡Department of EECS, Syracuse University, Syracuse, NY, 13244 USA
{ankitame,piyushb}@iitk.ac.in*, {oozdemir, adrozd}@androcs.com†,{emasazad,ckmohan,varshney}@syr.edu‡

***Abstract*—We consider the problem of joint admission control and power allocation in a multiband cognitive radio network (CRN) coexisting with multiple narrowband primary systems, and investigate two separate optimization problems: i) sum-rate maximization under primary user (PU) and secondary user (SU) quality of service (QoS) constraints; ii) sum-rate maximization and power minimization under PU and SU QoS constraints. We first show that these problems are NP-hard. Then we propose three different suboptimal algorithms for the first problem based on convex relaxation with tree pruning (CRTP), convex relaxation with gradual removal (CRGR) and genetic algorithms (GA). These algorithms offer different tradeoffs in terms of goodness of their solutions and computational complexity. For the second problem, we propose a multiobjective evolutionary algorithm which can generate a Pareto front in a time-efficient manner. Simulation results are provided to evaluate the proposed algorithms.**

***Index Terms*—Cognitive radio networks, resource allocation, optimization algorithms.**

I. Introduction

With the rapidly growing demand for wireless access, cognitive radio networking has become an active area of research. The concept of *cognitive radio network (CRN)* was proposed as a solution to maximize the efficiency of spectrum usage, where secondary (unlicensed or low-priority) users can access the licensed spectrum as long as they do not cause any “harmful” interference to the primary (licensed or high-priority) network. Two popular approaches for implementing cognitive radio networking are spectrum overlay and spectrum underlay [1]. In the spectrum overlay approach, secondary users (SUs) detect *spectrum holes* (frequency bands not used by primary users) by sensing the whole spectrum, and then transmit over them. In spectrum underlay approach, SUs coexist with the primary users (PUs) over a shared spectrum band as long as they don’t violate any PU quality-of-service (QoS) constraints.

In this paper, we focus on multiband CRNs utilizing spectrum underlay approach and investigate two optimization problems: i) Sum-rate maximization of the network under PU and SU QoS constraints; ii) Sum-rate maximization and power minimization of the network under PU and SU QoS constraints. We consider a scenario where there are a number of ongoing PU transmissions. The secondary network has a number of transmission requests which have some given QoS constraints. Secondary transmissions are not allowed to violate any existing PU QoS constraints. The problem is to select which secondary links to activate (admission control) and to find how to allocate power over the shared frequency band for those activated secondary links in the context of given optimization problems. Sum-rate maximization has been widely considered in the literature [2], [3], [4], [5] as a system optimization criterion, especially in the context of traditional wireless networks. In our first problem, given the QoS requirements of the PUs, each SU admission strategy yields a different solution region in terms of power variables and solution regions are disjoint. For an SU admission strategy, if there exists a power allocation strategy meeting QoS requirements of all the PUs and the admitted SUs simultaneously, then that solution region is deemed as feasible. To the best of our knowledge, this kind of multiband scenario with multiple joint requirements has not been considered in the cognitive radio literature. Similar to the sum-rate maximization problem, network power minimization has also been widely investigated. However, to the best of our knowledge, these two problems have not been investigated together in a multiobjective framework. This work serves as our attempt to introduce a network wide multiobjective optimization problem for sum-rate maximization and power minimization in CRNs, and propose multiobjective optimization algorithms to obtain the set of Pareto optimal solutions.

We show that the sum-rate maximization problem with QoS constraints is NP-hard, which implies that our second problem is also NP-hard. This result motivates the development of computationally efficient suboptimal algorithms. We propose three different algorithms to maximize sum-rate. The first algorithm iteratively attempts to find the feasible regions using a tree pruning approach. Then, in each feasible region, the solution is obtained by first approximating the original objective function (using a high signal-to-interference-plus-noise ratio (SINR) approximation) and then using a transformation that transforms the original nonconvex problem into a convex one. The second algorithm iteratively eliminates the infeasible links using a heuristic methodology where the algorithm first ignores the constraints in the original problem and uses a convex transformation as in the first algorithm, and then deletes the infeasible links using the constraints in the original problem. The third algorithm is based on a genetic algorithm (GA) which is a stochastic optimization algorithm widely used for NP-hard problems.

The second problem formulation is a multiobjective problem for which one needs to find a set of Pareto optimal solutions (trade-off solutions between two objectives) rather than a single solution. After finding the set of optimal solutions, it is up to the central controller to select one of these solutions. Multiobjective optimization problems can be solved by evolutionary algorithms such as non-dominated sorting genetic algorithm-II (NSGA-II) [6]. The original NSGA-II is unable to provide good solutions for our problem. Therefore, we modify the original NSGA-II algorithm to improve its performance. Simulation results are provided to show the performance of our modified algorithm.

The rest of the paper is organized as follows. In Section II, we introduce the system model and general problem statement. Sum-rate maximization problem is formulated in Section III, where we also present our proposed algorithms. In Section IV, we introduce our multiobjective optimization problem, i.e., sum-rate maximization and power minimization problem, and present the modified NSGA-II algorithm. We provide numerical results in Section V. Finally, concluding remarks and directions for future work are presented in Section VI.

II. System Model and Problem Statement

Consider a spectrum underlay CRN, where the available frequency band can be shared between SUs and PUs in the network as long as SU transmitters do not create harmful interference at PU receivers. We assume that the shared spectrum is divided into K discrete frequency subbands and, without loss of generality, each subband has an identical bandwidth of B Hz. This set of assumptions is applicable to systems using orthogonal-frequency-division-multiplexing (OFDM) technology which has been widely advocated to be a promising candidate technology for cognitive radio networks [7]. Our formulations are based on the physical model [8], which provides a realistic modeling of the physical communication environment by utilizing the path-loss model. The path loss between a transmitter-receiver pair n is given as $L_n = C/f^2 d_i^{\alpha}$, where C is a constant, f is the subband carrier, $(d_n > 0)$ is the distance between transmitter n and receiver n, and α is the attenuation constant. We assume that the path-loss in the received power is the dominant loss factor, and therefore, we neglect the effects of shadowing and multi-path fading.

For notational convenience, we number each secondary and primary transmitter-receiver pair by the indices $n \in \mathcal{N} = \{1, \ldots, N\}$ and $m \in \mathcal{M} = \{1, \ldots, M\}$, respectively, and refer to them as users. Throughout the paper, the terms subband and carrier are used interchangeably. We assume Gaussian channel with zero mean and variance N_0, and that the received interference is treated as white noise. Under these assumptions, the achievable data rate of user n can be expressed [9] as:

$$R_n = B \sum_{k=1}^{K} \log[1 + \gamma_n(k)],$$

where $\log$ is defined in base 2 and $\gamma_n(k)$ is the signal-to-interference-plus-noise ratio (SINR) of user n on carrier k,

$$\gamma_n(k) \triangleq \frac{p_n(k) L_n(k)}{N_0 + \sum_{l \in \mathcal{N} \cup \mathcal{M}, l \neq n} p_l(k) L_l(k)}. \quad (1)$$

The SINR condition for establishing a successful communication link n on carrier k is given by $\gamma_n(k) \geq \gamma_n^*$ [1].

Without loss of generality, we assume a narrowband primary network, where a single channel with predetermined transmit power values is allocated to each primary user. This scenario is applicable to networks where legacy radios have the licenses to operate on narrowband channels. Generalization to a wideband primary network is straightforward and does not affect the methodology. Secondary users utilize multiband techniques to access the spectrum and each secondary user has a power budget denoted by P^B.

The problem is to maximize the spectral efficiency of the secondary network while satisfying three sets of constraints: 1) SINR constraints at the primary receivers, 2) SINR constraints at the secondary receivers, 3) power budget constraints at the secondary transmitters. Next, we provide the details of our problem formulations and proposed algorithms.

III. Maximization of Sum-Rate Under QoS Constraints

Without loss of generality, we consider the worst case scenario, where the available frequency band is entirely occupied by primary users. We assume that each primary user occupies a single subband and they operate on disjoint subbands. This results in the equality $M = K$. These assumptions are made for notational convenience and they do not impact our formulations. Given primary network activity and location of users, the optimization problem is to maximize sum-rate (or achievable capacity) of the secondary network. The optimization variables are power levels allocated to each secondary user over each shared frequency subband.

Define $\mathbf{p}_n \triangleq [p_n(1), \ldots, p_n(K)]^T$ as the power allocation vector where each element represents the power level allocated to user n over each subband. User n is said to be inactive over frequency band k if $p_n(k) = 0$. A user is said to be active if it is transmitting on at least one subband. Let $\mathcal{F}_n$ denote the set of frequency channels with nonzero power allocations for session n, which implies $|\mathcal{F}_n| \leq K$. The notation $|\cdot|$ represents the cardinality of a set. In this case, our setting requires that $|\mathcal{F}_m| = 1$ for all $m \in \mathcal{P}$, and $\mathcal{F}_i \cap \mathcal{F}_j = \emptyset$ if $i \neq j$ and $i, j \in \mathcal{P}$. An SU is allowed to transmit on a carrier, if and only if it does not violate any SINR or power budget constraints.

We can now formulate the sum-rate maximization problem, referred to as P1, as follows:

$$\begin{aligned}
\text{Find} \quad & \mathbf{p}_n, \quad \forall n \in \mathcal{SU} \\
\text{Maximize} \quad & \sum_{n=1}^{N} R_n(\mathbf{p}_n) && (2) \\
\text{Subject to} \quad & \gamma_m(k) \geq \gamma_m^*, \quad \forall m \in \mathcal{M}, \quad \forall k \in \mathcal{F}_m, && (3) \\
& \gamma_n(k) \geq \mathbf{I}(p_n(k)) \gamma_n^*, \quad \forall n \in \mathcal{N}, \quad \forall k, && (4) \\
& p_n(k) \geq 0, \quad \forall n \in \mathcal{N}, \quad \forall k, && (5) \\
& \sum_{k=1}^{K} p_n(k) \leq P^B, \quad \forall n \in \mathcal{N}. && (6)
\end{aligned}$$

In P1, $\mathbf{I}(\cdot)$ is the indicator function for the set of positive real numbers. Inequality (3) represents a set of $K(= M)$ SINR constraints for the primary users. The inequality (4) represents a set of $N \times K$ SINR constraints for each of N SUs over each of K frequency subbands. The SU n will transmit at a frequency k, if and only if the SINR of that link is greater than or equal to the threshold SINR. If the link is not active over that subband, i.e., if $p_n(k) = 0$, the SINR constraint is automatically satisfied. It should be noted that P1 is in fact a *soft-spectrum allocation* and *power allocation* problem. The difference between conventional spectrum and power allocation formulations that have been considered in the literature, e.g. [10], [11], and our problem P1 is that

[1] γ^* is the minimum required SINR in order to maintain a certain quality-of-service (QoS), e.g., bit error rate (BER).

conventional formulations treat spectrum allocation as a hard allocation problem such that no two users share the same spectrum. In conventional settings, spectrum allocation is carried out first based on channel conditions followed by power allocation [10], [11]. However, in our formulation, the constraint in (4) provides a way to allocate the spectrum in such a way that multiple SUs can share the spectrum as long as their QoS constraints are not violated.

Proposition 1: The sum-rate maximization problem with QoS constraints P1 is NP-hard.

Proof: We prove this by restriction [12]. It was shown in [4] that the sum-rate maximization problem without the QoS constraints is NP-hard. The sum-rate maximization problem P1, which has the primary and secondary QoS constraints, includes the problem without the QoS constraints as a restricted special case. Specifically, P1 degenerates to the problem in [4] by setting $\gamma_n^* = \gamma_m^* = 0$ in (3) and (4). Therefore, P1 is also NP-hard. ∎

The NP-hardness of the above problem motivates the development of efficient suboptimal algorithms. In order to understand the problem better, let us consider a simple example of two SUs, one PU and one frequency subband. For a given set of locations, we draw the SU SINR constraints given in (4) in Figure 1(a). For now, we ignore the SINR constraints for PUs. The area inside the square formed by the bounds on the SU power budget is the region where power budget constraints are satisfied. There are four possibilities: both SUs transmit, only the first SU transmits, only the second SU transmits, or both SUs are off. When both SUs transmit, SINR constraints for both SUs must be satisfied simultaneously which is represented by the gray region in Figure 1(a). When only SU_a transmits, only the SINR constraint for SU_a needs to be satisfied. In that case, the constraint in (4) for SU_b is automatically satisfied because $p_b = 0$. Here, the dark line on the horizontal axis is feasible. Similar case holds when only SU_b transmits. When both are off, the constraints in (4) are automatically satisfied and this region is a single point $(0, 0)$. Now, with a different set of locations, it is possible that the receivers of both SUs are close to each other. Then the constraint region may look as shown in Figure 1(b). Here, there is no region where both SU SINR constraints for SUs and power budget constraints are satisfied simultaneously. In other words, it is not feasible for both SU_a and SU_b to transmit together. Another scenario could arise when some secondary transmitters and receivers are so far apart, or the receiver is so close to the PU transmitter such that there is no region when that user can be active, i.e, there is no region when the power budget as well as the SINR constraint for that SU is satisfied simultaneously. This scenario is shown in Figure 1(c). Note that, if this is the case, i.e., an SU cannot transmit even when other SU's are OFF, then the possibility of that SU transmitting when others are ON is automatically eliminated. We now consider the PU constraints defined in (3). For a given constant PU transmit power, the constraints in (3) define halfspaces. Therefore, in the above three scenarios, the addition of linear PU constraints will only reduce the volume of the feasible regions shown in Figures 1(a)-1(c) for a 2 dimensional case. Resulting feasible regions will be the intersection of the previous regions and the additional halfspaces.

We propose three different algorithms to solve the above problem based on the following methods: 1) convex relaxation with tree pruning; 2) convex relaxation with gradual removal; 3) genetic algorithms. The comparison of these three algorithms in terms of finding good solutions and time complexity will be investigated later. The following subsections provide detailed description of the algorithms.

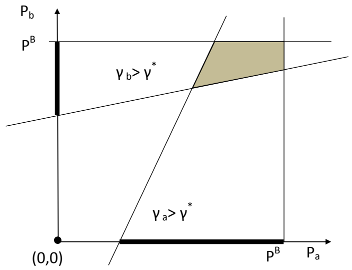
A. CRTP: Convex Relaxation with Tree Pruning

We start by noting that different combinations of ON and OFF power variables define different disjoint regions in space. Each of these regions differ from each other by order of dimensions. There are a total of $2^{N \times K}$ ON/OFF combinations and hence that many regions. However, the number of these regions which are feasible depends on the locations of secondary and primary users in the network.

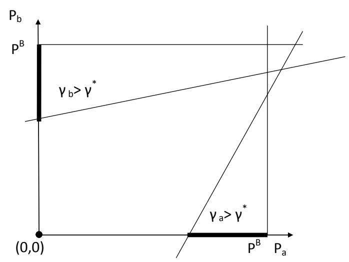
For each feasible region, there are M linear PU SINR constraints, N linear power budget constraints and some linear SU SINR constraints whose number depends upon the number of non zero elements of the power vector $\mathbf{p}_n$. Furthermore, we note that the objective function is nonconvex. In order to alleviate the nonconvexity issue, we first use the following approximation $\log(1+x) \approx \log(x)$ which is tight for values of $SINR \geq 5$. The same approximation has been made in [2], [5] for different problems. In a recent work [13], the authors have shown that $p_k = ce^{\mu s_k}$, $c, \mu > 0$ is the unique transformation that transforms the objective function defined by $g(\mathbf{p}) = \sum_{n=1}^{N} \log\left(\frac{p_n}{N_n + \sum_{i \neq n} a_{in} p_i}\right)$ into a convex function. Note that in $g(\centerdot)$, we set $K = 1$ for notational convenience and $a_{in} := L_i / L_n$. The same transformation has been used by others in similar formulations (e.g., [2], [14]). Using the high SINR approximation and applying the transformation $p_n = e^{s_n}$, the resulting problem becomes convex in each feasible region. However, the problem as a whole remains nonconvex as all these feasible regions are disjoint, resulting in a nonconvex global feasible set.

In order to solve this problem globally, we need to find the regions that are feasible. Investigating all the possible regions (2^{MN}) is a computationally expensive task especially for large number of variables. To reduce the computational effort, we observe that SU SINR constraints for a given region consist of a group of linear inequalities with number of variables equal to number of inequalities. We can consider these linear inequalities as equalities and solve for $\mathbf{p}_n$. The resulting solution, say (P^*), is the minimum feasible solution which satisfies (4). Once we have a (P^*) value for a given region we check to see if it satisfies M PU SINR constraints and also the N power budget constraints. If these constraints are violated at $p = P^*$, it means that there is no feasible p that satisfies the violated PU SINR and the power budget constraints without violating the SU SINR constraints. Using this reasoning, we can eliminate all the infeasible regions using a tree pruning method. We start with the lowest branch of the tree which has the smallest number of variables. Note that this branch defines a feasible region. We then solve the SU SINR linear equations for that region and then check if the solution violates the PU SINR or power budget constraints. If it violates any of these constraints, the region is eliminated and is not considered any further. We know that if a combination of some users active over some frequency band is infeasible then all combinations that are supersets of that combination are also infeasible. Then, we can prune that branch of the tree.

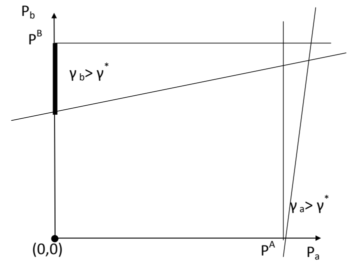
For each of the remaining feasible regions, we have a convex optimization problem which can be solved globally and



(a) Scenario-I



(b) Scenario-II



(c) Scenario-III

Fig. 1. Possible Scenarios in Secondary Power Allocations

TABLE I
COMPARISON OF NUMBER OF FEASIBLE REGIONS WITH PROBLEM SIZE

N	K	No. variables	No. Regions	Feasible Regions
4	3	12	4096	127
7	2	14	16384	375
8	2	16	65536	2144
9	2	18	262144	3356
10	2	20	1048576	55281

efficiently. The optimum solutions for each of these regions are stored and then compared to find the best solution, i.e., maximum sum-rate, and the corresponding power allocation vector.

If the SINR threshold γ_n^* is sufficiently large (i.e., $\log(1+x) \approx \log(x)$), CRTP algorithm will find solutions that are very close to global optimum since the relaxed convex problem approximates the original problem with high accuracy in the feasible regions. Note that the tree pruning approach helps to reduce the computational complexity, which can be a bottleneck for large problems with too many variables. Table I shows how the number of regions and the number of feasible regions scale with the problem size for a specific realization of the problem. The advantage we get from the tree pruning method can be deduced from the table.

B. CRGR - Convex Relaxation with Gradual Removal

We note that the complexity of the above CRPT algorithm is worst-case exponential. In order to find solutions faster, we propose a heuristic algorithm, referred to as CRGR, based on convex relaxation with gradual removal. This algorithm is very similar to the one proposed in [2] without the QoS constraints. Although this method does not guarantee an optimum solution, it can give good solutions with less computational effort.

Several different disjoint regions exist in the problem because of SU SINR constraints, which differ from region to region. Without SU SINR constraints, the remaining constraints define a space which is an intersection of halfspaces. This means the optimization space becomes a convex set. In CRGR, we use the same high SINR approximation as before ($\log(1+x) \approx \log(x)$) and apply the $p = e^s$ transformation. The resulting problem is convex. After solving this problem without the SU SINR constraints, we iteratively remove the links which violate the (SU and PU SINR) constraints the most, and also links for which the power level as a result of optimization is close to zero. This process is repeated until we get a solution which satisfies all the PU, SU and power budget constraints.

C. GA - Genetic Algorithm

Heuristic artificial intelligence techniques have been used to solve NP-hard problems which tend to exploit the structure of the problem and arrive at a good solution. In this section, we solve P1 with a genetic algorithm (GA) which belongs to the evolutionary class of artificial intelligence techniques. Following is the description of the GA employed in this paper.

1) Representation: A gene/solution is a structure comprising of $N \times K$ real variables corresponding to the powers of SUs. Let $\mathbf{p}_i^t$ be a solution with elements $p_n^t(k)$ at time t and $i \in \{1, \ldots, \mathcal{S}\}$ where $\mathcal{S}$ is the population size.

2) Initialization: The population is created by initializing each variable in each gene according to uniform random distribution between its lower and upper bounds.

Note that given the variable bounds $[0, p_{max}]$, it is very unlikely that some solutions have the form $p_n(k) = 0, \forall k$ where $\mathbf{p}_n = \mathbf{0}$ indicates the SU link is turned off and $\mathbf{p}_n \succcurlyeq \mathbf{0}$ denotes that the link n is active where $\succcurlyeq$ is the element-wise greater operator. In this form, GA fails to provide good solutions since it becomes hard to discover all the feasible regions. This fact motivates modifications that would increase the probability of creating individuals in each and every feasible region. Therefore, we change the variable bounds from $[0, p_{max}]$ to $[-p_{max}, p_{max}]$. Whenever a variable is less than zero, we treat it as equal to zero while evaluating the function. This modification provides more homogenous distribution for generating different types of SU admission strategies (some SU links are and on some of them off) and GA better explores the feasible regions.

3) Selection: The individuals that will go into the mating pool are chosen according to tournament selection [15].

4) Crossover: From the current population, $2 \times \mathcal{S}$ random pairs of individuals are created and the best of each pair is passed on into the mating pool. The individuals in the mating pool undergo Simulated Binary Crossover (SBX)[15] to form the offspring solutions. Let $\mathbf{p}_1^t$ and $\mathbf{p}_2^t$ be two parent solutions, then the offspring solutions are created according to,

$$\mathbf{p}_1^{t+1} = 0.5[(1+\beta_q)\mathbf{p}_1^t + (1-\beta_q)\mathbf{p}_1^t],$$
$$\mathbf{p}_2^{t+1} = 0.5[(1-\beta_q)\mathbf{p}_2^t + (1+\beta_q)\mathbf{p}_2^t],$$

where β_q follows the probability distribution as defined in [15].

In order to promote diversity, the negative elements of $\mathbf{p}_1^t$ and $\mathbf{p}_2^t$ are first updated by a new value which is uniformly generated within the interval $[-p_{max}, 0]$. Then the SBX operation defined above is performed.

5) Mutation: If $p_n^t(k) > 0$, polynomial mutation is performed [15]. If $p_n^t(k) < 0$ then any of the following mutations is performed: (i) $p_n^t(k) = rand[0; pmax]$. This means, if we mutate a variable which is OFF, we turn it ON and assign it a random power. (ii) The user is turned ON at a power at which it just satisfies its SINR constraint. (iii) The user is turned ON at a power which is equal to the average of its power at other channels.

6) Elitism: All solutions of generation t and $t-1$ are combined and the best solutions from this set go to population of $t+1$.

7) Evaluate: Repeat Steps 2 through 6 until the solution does not improve significantly, i.e., the best function value does not improve by more than 0.1 percent, for some number of iterations.

IV. Maximization of Sum-Rate and Minimization of Network Power Under QoS Constraints

In this section, we consider joint minimization of total power consumption and maximization of sum-rate of the CRN. Minimizing total network power is an important problem especially when the network has a constraint on the interference it can cause to other neighboring networks. Our bi-objective optimization problem referred to as P2 is defined as follows.

$$\begin{aligned} &\text{Find} && \mathbf{p}_n, \quad \forall n \in \mathcal{SU} \\ &\text{Minimize} && \left\{ -\sum_{n=1}^{N} R_n(\mathbf{p}_n) \right\}, \left\{ \sum_{n=1}^{N}\sum_{k=1}^{K} p_n(k) \right\} && (7) \\ &\text{Subject to} && \text{Constraints of P1} && (8) \end{aligned}$$

P2 is a multiobjective optimization problem (MOP) for which one needs to find a set of Pareto optimal (trade-off) solutions (non-dominated by any other solution) between the two objectives. After finding the set of trade-off solutions, final decision is made by the network controller to select one of these solutions. A well-known technique for solving MOPs is to minimize a weighted sum of the objectives. However, minimizing the weighted sum of the objectives suffers from several drawbacks. As an example, a uniform spread of weights rarely produces a uniform spread of points on the Pareto front hence the entire Pareto-optimal front can not be exploited. For this problem, we use a modified version of NSGA-II [6], which is a state of the art multiobjective evolutionary algorithm. Initially, population of size $\mathcal{S}$ is duplicated by crossover and mutation operations which are explained in the previous section. NSGA-II is an elitist algorithm which uses a non-domination ranking approach where solutions in the duplicated population are ranked according to fronts they belong to. For instance, a solution belongs to the first front, if no other solution in the population dominates it. Similarly, the second front is composed of only the solutions that are dominated by the first front and so on. After several iterations, the NSGA-II population consist of non-dominated solutions which belong to the Pareto-optimal front.

V. Numerical Results

In this section, we provide numerical results to evaluate the performance of our proposed algorithms. The transmit power of each primary transmitter and the power budget P^B for each secondary transmitter are set to 6 dB and -3 dB, respectively. The SINR thresholds for PUs and SUs are $\gamma_m^* = 20$ dB and $\gamma_n^* = 10$ dB for $n = 1, \ldots, N$ and $m = 1, \ldots, M$, respectively. We consider an area of 5×5 kilometers. PU and SU transmitters are randomly deployed in the area. PU receivers are randomly deployed within ζ_k distance of their respective transmitters, where ζ_k is chosen to provide 20 dB at the boundary of their deployment region. Note that we

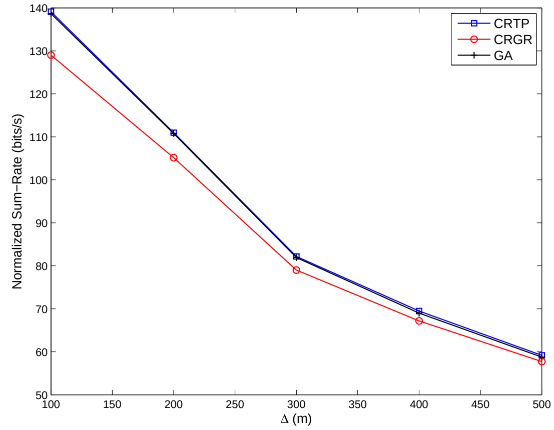


Fig. 2. Performance of Proposed Algorithms for P1 (N=M=K=3)

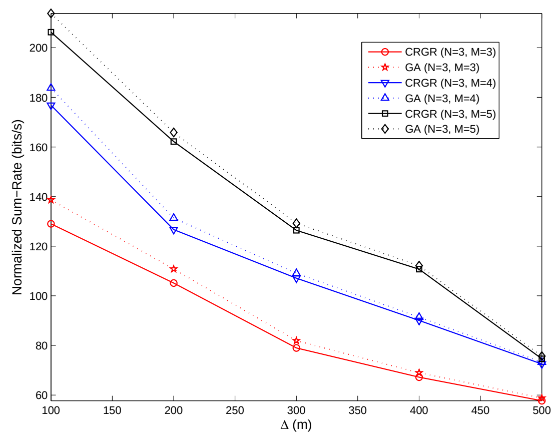


Fig. 3. Performance of CRGR versus GA for P1

set $M = K$ as explained in Section III, i.e, the number of subbands is equal to the number of primary users. Each subband has a bandwidth of $B = 6$ MHz. Attenuation constant $\alpha = 4$. SU receivers are randomly deployed within Δ distance of their respective transmitters.

We vary Δ from 100 meters to 500 meters and provide sum-rate results averaged over 100 Monte Carlo simulations for each Δ value. Figure 2 depicts normalized sum-rate results, i.e., $\frac{1}{B}\sum_n R_n$, obtained for the case when $N = M = K = 3$. As expected, the optimum sum-rate values decrease as Δ increases, because high Δ values imply low SINR at the secondary receivers. Since SU SINR thresholds are selected to be $\gamma_n^* = 10$ dB, one should note that the function approximation $\log(1 + SINR) \approx \log(SINR)$ is highly accurate in feasible regions. Therefore, CRTP should provide solutions that are very close to the global optimum. In this case, the solutions obtained by CRTP can be used as benchmarks for those obtained by CRGR and GA. As is clear from Figure 2, GA performs very close to CRTP. Among the three algorithms, CRGR performs the worst in terms of maximizing sum-rate. The complexity of the CRTP algorithm is worst-case exponential, which can be computationally prohibitive for large problems. In contrast, the CRGR algorithm has the lowest computational complexity because of its greedy nature. Therefore, it is scalable for larger problems with high number of users and frequency bands. GA provides a balance between optimality and computational complexity, since it can provide

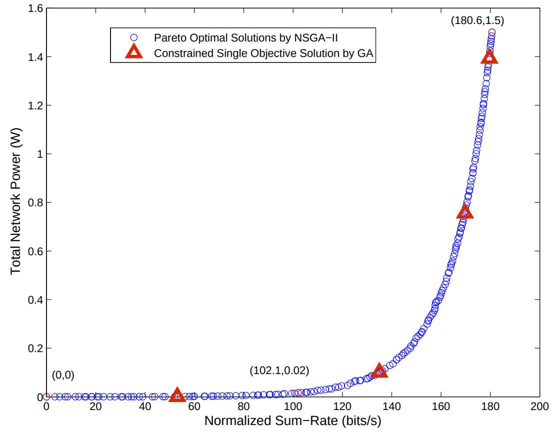


Fig. 4. Pareto Optimal Fronts for P2

solutions that are very close to that of the CRTP with reduced computational complexity. In Figure 3, we compare sum-rate solutions obtained by CRGR and GA for different sizes of shared frequency bands. It is clear from the figure that as K increases, i.e., as the shared bandwidth gets larger, the maximum sum-rate increases as expected. Furthermore, the performance of the CRGR algorithm is very close to that of the GA.

Figure 4 shows the Pareto front containing 200 non-dominated solutions for a particular realization of P2 when $N = 3$ and $M = K = 4$. The Pareto front is obtained after after 200,000 function evaluations. A particular solution that belongs to the Pareto front is a solution to our first problem with one additional constraint on the total network power. Therefore, we can verify the Pareto optimal solutions by selecting some test points on the Pareto front by constraining the total network power and maximizing sum-rate through GA developed for P1. These solutions are denoted by triangle points on Figure 4. It is interesting to see from Figure 4 that the normalized sum-rate can be increased from 0 to about 100 bits/s with only an incremental increase (0.02 Watts) in the total network power. The Pareto front obtained by our modified NSGA-II algorithm helps the network controller clearly see the trade offs between different solutions. The advantage of formulating multiobjective optimization problems for CRNs is clear in this example.

VI. Conclusion

We investigated two optimization problems in multiband cognitive radio networks with PU and SU QoS constraints: i) sum-rate maximization, ii) sum-rate maximization and power minimization. Optimization variables are activation of links and power values allocated over the shared frequency band. Motivated by the fact that these problems are NP-hard, we developed three suboptimal algorithms based on convex relaxation with tree pruning (CRTP), convex relaxation with gradual removal (CRGR) and genetic algorithms (GA). Each algorithm offers different trade offs in optimality and computational complexity. CRTP provides the best performance in terms of optimality; however, it is computationally the most expensive algorithm. CRGR offers the lowest computational complexity; however it suffers from high degree of suboptimality. GA provides a balance between optimality and computational complexity. For the second problem, we developed an efficient multiobjective optimization algorithm based on the modified NSGA-II. Simulations results demonstrated that our algorithms provide excellent performance and offer various alternative solutions to the network controller.

In the future, we will extend our formulations to other communication scenarios in cognitive radio networks. Such scenarios include heterogeneous QoS constraints for different users such as combinations of data rate and SINR constraints, and other objective functions such as maximizing number of active users.

Acknowledgment

This material is based on research sponsored in part by the Air Force Research Laboratory, under agreement number FA8750-10-C-0221, and CASE: The Center for Advanced Systems and Engineering, a NYSTAR center for advanced technology at Syracuse University. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Research Laboratory or the U.S. Government.

References

[1] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.

[2] M. Johansson, L. Xiao, and S. Boyd, "Simultaneous routing and power allocation in CDMA wireless data networks," in *Proc. Int. Conf. Communications*, Anchorage, AK, May 2003.

[3] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicenced bands," *IEEE J. Sel. Areas in Comm.*, vol. 25, no. 3, pp. 517–528, Apr. 2007.

[4] Z. Luo and S. Zhang, "Dynamic spectrum management: complexity and duality," *IEEE J. Sel. Topics Signal Process., Special Issue on Signal Process. Netw. Dyn. Spectrum Access*, vol. 2, no. 1, pp. 57–73, Feb. 2008.

[5] J. Tadrous, A. Sultan, and M. Nafie, "Admission and power control in spectrum sharing cognitive radio networks," *IEEE Trans. Wireless Comm.*, vol. 10, no. 6, pp. 1945–1955, Jun. 2011.

[6] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evolutionary Comp.*, vol. 6, no. 2, pp. 182–197, Apr. 2002.

[7] R. Rajbanshi, A. M. Wyglinski, and G. J. Minden, *OFDM-Based Cognitive Radios for Dynamic Spectrum Access Networks. Cognitive Radio Communications Networks (Vijay Bhargava and Ekram Hossain, Eds.).* Springer-Verlag, 2007.

[8] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Information Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[9] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley, 1991.

[10] P. Wang, M. Zhao, L. Xiao, S. Zhou, and J. Wang, "Power allocation in OFDM-based cognitive radio systems," in *Proc. IEEE Global Communications Conference*, Nov. 2007.

[11] G. Miao, N. Himayat, G. Y. Li, and S. Talwar, "Low-complexity energy-efficient scheduling for uplink OFDMA," *IEEE Trans. Communications*, vol. 60, no. 1, pp. 112–120, Jan. 2012.

[12] M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman and Company, 1979.

[13] H. Boche, S. Naik, and T. Alpcan, "Characterization of convex and concave resource allocation problems in interference coupled wireless systems," *IEEE Trans. Signal Process.*, vol. 59, no. 5, pp. 2382–2394, May 2011.

[14] D. Julian, M. Chiang, D. O'Neill, and S. Boyd, "QoS and fairness constrained convex optimization of resource allocation for wireless cellular and ad hoc networks," in *Proc. IEEE Intl. Conf. Computer Communications (INFOCOM'02)*, Jun. 2002.

[15] K. Deb, *Multi-objective Optimization Using Evolutionary Algorithms*. Wiley, 2001.